

Online Appendix

A.1 Proofs of the Results in Section 3

Proof of Proposition 1

In the second stage of the game, λ and I are fixed. From (1), the manager's first order condition, $\partial EUM/\partial \delta = 0$, can be restated as:

$$\left[e^{-x}\lambda - (1 - e^{-x})b \right] (1 + g)I = 0.$$

Hence, in equilibrium:

$$(\lambda + b)e^{-x} = b. \quad (\text{A-1})$$

By rearranging (A-1), I obtain the auditor's best response:

$$x^* = \log \frac{\lambda + b}{b}. \quad (\text{A-2})$$

Next, from (4), the auditor's first order condition, $\partial EUA/\partial x = 0$, is restated as:

$$-kx + e^{-x}(a + v)\delta = 0. \quad (\text{A-3})$$

By rearranging (A-3), I obtain the manager's best response:

$$\begin{aligned} \delta^* &= \frac{kxe^x}{a + v} \\ &= \frac{\lambda + b}{(a + v)b} k \log \frac{\lambda + b}{b}, \end{aligned} \quad (\text{A-4})$$

where the second equality follows because of (A-2).

Proof of Lemma 1

Differentiating the solutions in (A-2) and (A-4) with respect to v yields:

$$\frac{\partial x}{\partial v} = 0, \text{ and} \quad (\text{A-5})$$

$$\frac{\partial \delta}{\partial v} = -\frac{(\lambda + b)k}{(a + v)^2 b} \log \frac{\lambda + b}{b} < 0. \quad (\text{A-6})$$

These results in (A-5) and (A-6) lead to (1) and (2) of Lemma 1, respectively.

Proof of Proposition 2

From (6), the probability of detection in equilibrium is:

$$\begin{aligned}
 q(x^*) &= 1 - e^{-x^*} \\
 &= 1 - \frac{b}{\lambda + b} \\
 &= \frac{\lambda}{\lambda + b}.
 \end{aligned}$$

In addition, from (6), the manager's expected payoff, (1), is restated as:

$$\begin{aligned}
 EUM &= \left[\frac{b}{\lambda + b}(1 - \lambda + \delta\lambda) + \frac{\lambda}{\lambda + b}(1 - \lambda - b\delta) \right] (1 + g)I \\
 &= \frac{1}{\lambda + b}(b - b\lambda + b\delta\lambda + \lambda - \lambda^2 - b\delta\lambda)(1 + g)I \\
 &= (1 - \lambda)(1 + g)I.
 \end{aligned}$$

Proof of the first-stage equilibrium condition

From the investor's expected payoff (2), the equilibrium condition (3), and the assumption $i = 0$:

$$\begin{aligned}
 I_M &= EUV \\
 &= \left[(1 - q(x))(1 - \delta)\lambda + q(x)\lambda \right] (1 + g)I \\
 &= \lambda \left[1 - (1 - q(x))\delta \right] (1 + g)I.
 \end{aligned}$$

Recall $I = I_M + W - F$, or $I_M = I - W + F$. Then:

$$\begin{aligned}
 I^* &= \frac{W - F^*}{1 - \lambda(1 + g)[1 - (1 - q(x^*))\delta^*]} \\
 &= \frac{W - F^*}{1 - \lambda(1 + g)\left(1 - e^{-x^*} \frac{kx^* e^{x^*}}{a + v}\right)} \\
 &= \frac{W - F^*}{1 - \lambda(1 + g)\left(1 - \frac{kx^*}{a + v}\right)}.
 \end{aligned}$$

About the audit fee and PA (4)

From the auditor's zero profit condition, (5), I can state the equilibrium audit fee as:

$$\begin{aligned}
 F^* &= C(x^*) + a\delta^*(1 - q(x^*)) - v\delta q(x^*) \\
 &= \frac{k}{2}x^{*2} + a \frac{kx^* e^{x^*}}{a + v} e^{-x^*} - v \frac{kx^* e^{x^*}}{a + v} (1 - e^{-x^*}) \\
 &= \frac{k}{2}x^{*2} + \frac{akx^*}{a + v} - v \frac{kx^* e^{x^*}}{a + v} + \frac{vkx^*}{a + v} \\
 &= \frac{k}{2}x^{*2} + kx^* - \frac{vkx^* e^{x^*}}{a + v}.
 \end{aligned}$$

In equilibrium, the first derivative of $F(x)$ is:

$$\begin{aligned}
 F'(x) &= kx + k - \frac{vk}{a+v}(e^x + xe^x) \\
 &= k(x+1) \left(1 - \frac{v}{a+v}e^x \right).
 \end{aligned}$$

Note $k > 0$ and $x > 0$. Then $F'(x) > 0$ holds if and only if $e^x < (a+v)/v$, or equivalently:

$$\lambda < \frac{ab}{v}. \quad (\text{A-7})$$

I compare the audit fee, F , and the audit cost, C :

$$\begin{aligned}
 F - C(x) &= \frac{k}{2}x^2 + kx - \frac{vkxe^x}{a+v} - \frac{k}{2}x^2 \\
 &= kx \left(1 - \frac{ve^x}{a+v} \right) \\
 &= \frac{kvx}{a+v} \left(\frac{a+v}{v} - e^x \right).
 \end{aligned}$$

Hence, $F > C$ holds if, and only if, the inequality (A-7) holds. Because the manager chooses λ between 0 and 1, I can ensure that (A-7) always holds by assuming:

$$1 < \frac{ab}{v},$$

which constitutes PA (4) in Section 2.

Preparation for the comparative analysis

Recall that in the equilibrium of the game, the equality (11) holds and the optimal level of I is determined as in (10). I let D denote the denominator of (10):

$$D \equiv 1 - \lambda(1+g) \left(1 - \frac{kx}{a+v} \right).$$

Later, I prove that PAs ensure $D > 0$ and $I > 0$. These inequalities together ensure that the numerator of (10) is positive.

Next, I focus on the condition (11). Note:

$$\frac{dI}{d\lambda} = \frac{\partial I}{\partial \lambda} + \frac{\partial I}{\partial x} \frac{dx}{d\lambda}. \quad (\text{A-8})$$

I consider the terms on the right hand side of (A-8): $\partial I/\partial \lambda$, $\partial I/\partial x$, and $dx/d\lambda$. First,

$$\begin{aligned}
 \frac{\partial I}{\partial \lambda} &= \frac{-\frac{\partial F}{\partial \lambda} D - \frac{\partial D}{\partial \lambda} (W - F)}{D^2} \\
 &= \frac{-\frac{\partial D}{\partial \lambda} (W - F)}{D^2} \\
 &= (1+g) \left(1 - \frac{kx}{a+v} \right) \frac{W - F}{D^2},
 \end{aligned}$$

where the second equality holds because $F(x)$ is not directly related to λ . In addition,

$$\begin{aligned} \frac{\partial I}{\partial x} &= \frac{-\frac{\partial F}{\partial x}D - \frac{\partial D}{\partial x}(W - F)}{D^2} \\ &= \frac{-\frac{\partial F}{\partial x}D - \lambda(1 + g)\frac{k}{a + v}(W - F)}{D^2}, \text{ and} \\ \frac{dx}{d\lambda} &= \frac{1}{\lambda + b} \frac{1}{b} \\ &= \frac{1}{\lambda + b}. \end{aligned}$$

Then, (11) can be restated as follows:

$$\begin{aligned} (1 - \lambda) \left[\frac{(1 + g)\left(1 - \frac{kx}{a + v}\right)(W - F)}{D^2} \right. \\ \left. + \frac{1}{\lambda + b} \frac{-F'D - \lambda(1 + g)\frac{k}{a + v}(W - F)}{D^2} \right] - \frac{W - F}{D} = 0. \end{aligned} \tag{A-9}$$

Multiplying both sides of (A-9) by $D^2/(W - F) > 0$ yields:

$$(1 - \lambda) \left[(1 + g) \left(1 - \frac{kx}{a + v}\right) + \frac{1}{\lambda + b} \left\{ -\frac{F'D}{W - F} - \lambda(1 + g)\frac{k}{a + v} \right\} \right] - D = 0. \tag{A-10}$$

Rearranging the left-hand side (LHS) of (A-10) yields:

$$\begin{aligned} (LHS) &= (1 - \lambda)(1 + g)\left(1 - \frac{kx}{a + v}\right) + \frac{1 - \lambda}{\lambda + b} \left[-\frac{F'D}{W - F} - \lambda(1 + g)\frac{k}{a + v} \right] - D \\ &= (1 - \lambda)(1 + g)\left(1 - \frac{kx}{a + v}\right) - \frac{1 - \lambda}{\lambda + b} \lambda(1 + g)\frac{k}{a + v} - D - \frac{1 - \lambda}{\lambda + b} \frac{F'D}{W - F} \\ &= (1 - \lambda)(1 + g)\left(1 - \frac{kx}{a + v}\right) - \frac{1 - \lambda}{\lambda + b} \lambda(1 + g)\frac{k}{a + v} \\ &\quad - 1 + \lambda(1 + g)\left(1 - \frac{kx}{a + v}\right) - \frac{1 - \lambda}{\lambda + b} \frac{F'D}{W - F} \\ &= (1 + g)\left(1 - \frac{kx}{a + v}\right) - \frac{1 - \lambda}{\lambda + b} \lambda(1 + g)\frac{k}{a + v} - 1 - \frac{1 - \lambda}{\lambda + b} \frac{F'D}{W - F} \\ &= g - (1 + g)\frac{kx}{a + v} - \frac{1 - \lambda}{\lambda + b} \lambda(1 + g)\frac{k}{a + v} - \frac{1 - \lambda}{\lambda + b} \frac{F'D}{W - F} \\ &= g - \frac{k(1 + g)}{a + v} \left\{ x + \lambda \frac{1 - \lambda}{\lambda + b} \right\} - \frac{1 - \lambda}{\lambda + b} \frac{F'D}{W - F} \\ &= g - \frac{k(1 + g)}{a + v} \left\{ x + \lambda \frac{1 - \lambda}{\lambda + b} \right\} - \frac{1 - \lambda}{\lambda + b} \frac{F'}{W - F} \left[1 - \lambda(1 + g)\left(1 - \frac{kx}{a + v}\right) \right]. \end{aligned} \tag{A-11}$$

I let $H(\lambda)$ denote the expression in (A-11). Hence, the equilibrium condition (11) is equivalent to $H(\lambda) = 0$.

A.2 Proofs of the Parameter Assumptions

I prove that PAs in Section 2 are sufficient for precluding corner solutions and hence for doing comparative analysis.

Lemma 2 *PA (1) to (3) in Section 2 constitute sufficient conditions for the following inequalities (1) to (4) to hold:*

- 1) $\delta^* < 1$
- 2) $\left[\frac{dEUM}{d\lambda} \right]_{\lambda=0} > 0$, or equivalently, $H(0) > 0$,
- 3) $\left[\frac{dEUM}{d\lambda} \right]_{\lambda=1} < 0$, or equivalently, $H(1) < 0$, and
- 4) $D > 0$.

Under (2) and (3) in Lemma 2, there is $\lambda \in (0, 1)$ that satisfies the sufficient condition for EUM to take its local maximum:

$$\frac{dEUM}{d\lambda} = 0 \quad \text{and} \quad \frac{d^2 EUM}{d\lambda^2} < 0. \quad (\text{A-12})$$

Recall that this condition (A-12) can be restated as as:

$$H = 0 \quad \text{and} \quad \frac{dH}{d\lambda} < 0. \quad (\text{A-13})$$

Furthermore, Proposition 2 ensures $I^* > 0$. If the manager chooses λ close to 0 in the first stage of the game, x^* and F^* approach 0 from (6) and (8). Even in this extreme case, I and EUM are positive from PA (4), (10) and (9). Hence, in his optimal choice, the manager chooses $I^* > 0$.

Proof of Lemma 2

To verify (1) in Lemma 2, recall (7). The equilibrium δ requires:

$$\delta^* = \frac{\lambda + b}{(a + v)b} k \log \frac{\lambda + b}{b} < 1.$$

By rearranging this inequality, I obtain:

$$k < \frac{(a + v)b}{(\lambda + b) \log \frac{\lambda + b}{b}}. \quad (\text{A-14})$$

I later verify λ is in $(0, 1)$. Hence, the inequality in (A-14) holds by restricting k as follows:

$$k < \frac{(a + v)b}{(1 + b) \log \frac{1 + b}{b}}.$$

This is PA (1) in Section 2.

To verify (2) in Lemma 2, I note:

$$H(0) = g - \frac{1}{b} \frac{k - \frac{vk}{a+v}}{W}.$$

Then, $H(0) > 0$ is equivalent to:

$$W > \frac{k}{bg} \frac{a}{a+v}.$$

This is PA(2) in Section 2.

To verify (3) and (4) in Lemma 2, I note:

$$\begin{aligned} H(1) &= g - \frac{k(1+g)}{a+v} x \\ &= -D(\lambda = 1). \end{aligned}$$

Hence:

$$H(1) < 0 \iff D(\lambda = 1) > 0. \quad (\text{A-15})$$

In addition, I note:

$$D(\lambda = 0) = 1 > 0. \quad (\text{A-16})$$

I consider the minimum of D for λ in $[0, 1]$.

[1] If D takes its minimum at λ in $(0, 1)$:

First, I note:

$$\begin{aligned} D &= 1 - \lambda(1+g)\left(1 - \frac{kx}{a+v}\right) \\ &= 1 - \lambda(1+g)\left(1 - \frac{k}{a+v} \log \frac{\lambda+b}{b}\right). \end{aligned} \quad (\text{A-17})$$

The first and second derivatives of D with respect to λ are:

$$\begin{aligned} \frac{dD}{d\lambda} &= -(1+g) \left[\left(1 - \frac{k}{a+v} \log \frac{\lambda+b}{b}\right) + \lambda \left(-\frac{k}{a+v} \frac{b}{\lambda+b} \frac{1}{b}\right) \right] \\ &= -(1+g) \left[1 - \frac{k}{a+v} \log \frac{\lambda+b}{b} - \frac{k}{a+v} \frac{\lambda}{\lambda+b} \right]. \\ \frac{d^2D}{d\lambda^2} &= -(1+g) \left[-\frac{k}{a+v} \frac{b}{\lambda+b} \frac{1}{b} - \frac{k}{a+v} \frac{(\lambda+b) - \lambda}{(\lambda+b)^2} \right] \\ &= -(1+g) \left[-\frac{k}{a+v} \frac{1}{\lambda+b} - \frac{k}{a+v} \frac{b}{(\lambda+b)^2} \right] \\ &= (1+g) \frac{k}{a+v} \left(\frac{1}{\lambda+b} + \frac{b}{(\lambda+b)^2} \right) > 0 \end{aligned} \quad (\text{A-18})$$

I consider the minimum of D . If the first order condition is met, the expression in (A-18) is set equal to 0. Then:

$$1 - \frac{k}{a+v} \log \frac{\lambda+b}{b} = \frac{k}{a+v} \frac{\lambda}{\lambda+b}. \quad (\text{A-19})$$

This equality (A-19) implies that the minimum of D in (A-17) is:

$$\begin{aligned} D_{\min} &= 1 - \lambda(1+g) \frac{k}{a+v} \frac{\lambda}{\lambda+b} \\ &= 1 - \lambda^2(1+g) \frac{k}{a+v} \frac{1}{\lambda+b}. \end{aligned}$$

Then $D_{\min} > 0$ is equivalent to:

$$1 > \lambda^2(1+g) \frac{k}{a+v} \frac{1}{\lambda+b}. \quad (\text{A-20})$$

By rearranging (A-20), I obtain:

$$\begin{aligned} g &< \frac{1}{\lambda^2 \frac{k}{a+v} \frac{1}{\lambda+b}} - 1 \\ &= (a+v) \frac{\lambda+b}{\lambda^2 k} - 1 \\ &= \frac{a+v}{k} \left(\frac{1}{\lambda} + \frac{b}{\lambda^2} \right) - 1. \end{aligned}$$

Hence, the condition:

$$\begin{aligned} g &< \frac{a+v}{k} \left(\frac{1}{1} + \frac{b}{1^2} \right) - 1 \\ &= \frac{(a+v)(b+1)}{k} - 1, \end{aligned}$$

ensures (4) in Lemma 2. Since D is always positive under this condition, (A-15) leads to $H(1) < 0$. Then, (3) in Lemma 2 holds.

[2] If D takes its minimum at $\lambda = 0$:

In this case, the inequality (A-16) implies $D > 0$ for any λ in $[0, 1]$. Hence, (4) in Lemma 2 holds. In addition, from (A-15), (3) in Lemma 2 is also true.

[3] If D takes its minimum at $\lambda = 1$:

$D(\lambda = 1) > 0$ suffices to ensure (4) in Lemma 2. In this case, (3) in Lemma 2 is immediate from (A-15). Hence, I impose:

$$D(\lambda = 1) = 1 - (1+g) \left(1 - \frac{k}{a+v} \log \frac{1+b}{b} \right) > 0. \quad (\text{A-21})$$

Note that rearranging PA (1) gives:

$$\frac{k}{a+v} \log \frac{1+b}{b} < \frac{b}{1+b}. \quad (\text{A-22})$$

This inequality (A-22) implies:

$$1 - \frac{k}{a+v} \log \frac{1+b}{b} > \frac{1}{1+b} > 0. \tag{A-23}$$

By rearranging the inequality in (A-21), I obtain:

$$g < \frac{1}{1 - \frac{k}{a+v} \log \frac{1+b}{b}} - 1. \tag{A-24}$$

From the discussion in [1] to [3], the constraints:

$$\left\{ \begin{aligned} g &< \frac{(a+v)(b+1)}{k} - 1 && \text{(A-25)} \\ g &< \frac{1}{1 - \frac{k}{a+v} \log \frac{1+b}{b}} - 1 && \text{(A-26)} \end{aligned} \right.$$

constitute PA (3) in Section 2.

A.3 Proofs of the Results in Section 4

Proof of Proposition 3

From the implicit function theorem, I obtain:

$$\frac{d\lambda}{dv} = - \frac{\frac{\partial H}{\partial v}}{\frac{\partial H}{\partial \lambda}}. \tag{A-27}$$

From the manager's second order condition, the following inequality holds (see [A-13], noting I use the different differential notations for clarity):

$$\frac{\partial H}{\partial \lambda} < 0. \tag{A-28}$$

In addition, from (A-11), I obtain:

$$\begin{aligned} \frac{\partial H}{\partial v} &= \frac{k(1+g)}{(a+v)^2} \left[x + \frac{\lambda(1-\lambda)}{\lambda+b} \right] \\ &\quad - \frac{1-\lambda}{\lambda+b} \left[\frac{\partial F'}{\partial v} \frac{D}{W-F} + \frac{F'D}{(W-F)^2} \frac{\partial F}{\partial v} + \frac{F'}{W-F} \frac{\partial D}{\partial v} \right], \end{aligned} \tag{A-29}$$

where:

$$\begin{aligned} \frac{\partial F'}{\partial v} &= - \left[k(a+v)^{-1} + vk(-1)(a+v)^{-2} \right] (e^x + xe^x) \\ &= -k(e^x + xe^x) \frac{a+v-v}{(a+v)^2} \\ &= -\frac{ak(e^x + xe^x)}{(a+v)^2} < 0, \\ \frac{\partial F}{\partial v} &= -\frac{kxe^x(a+v) - vkxe^x}{(a+v)^2} \\ &= -\frac{akxe^x}{(a+v)^2} < 0, \end{aligned} \tag{A-30}$$

$$\begin{aligned}\frac{\partial D}{\partial v} &= \lambda(1+g)(-1)\frac{kx}{(a+v)^2} \\ &= -\frac{\lambda(1+g)kx}{(a+v)^2} < 0.\end{aligned}\tag{A-31}$$

Hence, the expression (A-29) is positive. The total derivative in (A-27) is then positive.

Proof of Proposition 4

I express the derivative dI/dv as in (12). From (11), where I use the different differential notation for clarity, I obtain:

$$\frac{\partial I}{\partial \lambda} = \frac{I}{1-\lambda} > 0.$$

From Proposition 3, $d\lambda/dv$ is positive. In addition,

$$\begin{aligned}\frac{\partial I}{\partial v} &= \frac{\frac{\partial(W-F)}{\partial v}D - (W-F)\frac{\partial D}{\partial v}}{D^2} \\ &= \frac{-\frac{\partial F}{\partial v}D - (W-F)\frac{\partial D}{\partial v}}{D^2},\end{aligned}\tag{A-32}$$

where, from (A-30) and (A-31), the partial derivatives in the numerator of (A-32) are both negative.

Hence, the expression (A-32) is positive. The expression (12) is then positive.

Proof of Proposition 5

Differentiating x with respect to v shows:

$$\begin{aligned}\frac{dx}{dv} &= \frac{d \log \frac{\lambda+b}{b}}{dv} \\ &= \frac{b}{\lambda+b} \frac{d \frac{\lambda+b}{b}}{dv} \\ &= \frac{b}{\lambda+b} \frac{d\lambda}{dv} \\ &= \frac{d\lambda}{\lambda+b} > 0,\end{aligned}$$

where the final inequality follows from Proposition 3.

Proof of Proposition 6

Differentiating EUM with respect to v yields:

$$\begin{aligned}\frac{dEUM}{dv} &= (1+g) \left[-\frac{d\lambda}{dv} I + (1-\lambda) \frac{dI}{dv} \right] \\ &= (1+g) \left[-\frac{d\lambda}{dv} (1-\lambda) \frac{dI}{d\lambda} + (1-\lambda) \frac{dI}{dv} \right] \\ &= (1+g)(1-\lambda) \left(\frac{dI}{dv} - \frac{d\lambda}{dv} \frac{dI}{d\lambda} \right) \\ &= (1+g)(1-\lambda) \frac{\partial I}{\partial v} > 0,\end{aligned}$$

where the final inequality follows from (A-32).