Online Appendix

A.1 Proofs of the Results in Section 3

Proof of Proposition 1

In the second stage of the game, λ and I are fixed. From (1), the manager's first order condition, $\partial EUM/\partial\delta = 0$, can be restated as:

$$\Big[e^{-x}\lambda - (1-e^{-x})b\Big](1+g)I = 0.$$

Hence, in equilibrium:

$$(\lambda + b)e^{-x} = b. \tag{A-1}$$

By rearranging (A-1), I obtain the auditor's best response:

$$x^* = \log\frac{\lambda+b}{b}. \tag{A-2}$$

Next, from (4), the auditor's first order condition, $\partial EUA/\partial x = 0$, is restated as:

$$-kx + e^{-x}(a+v)\delta = 0. \tag{A-3}$$

By rearranging (A-3), I obtain the manager's best response:

$$\begin{aligned}\delta^* &= \frac{kxe^x}{a+v}\\ &= \frac{\lambda+b}{(a+v)b}k\log\frac{\lambda+b}{b},\end{aligned} \tag{A-4}$$

where the second equality follows because of (A-2).

Proof of Lemma 1

Differentiating the solutions in (A-2) and (A-4) with respect to v yields:

$$\frac{\partial x}{\partial v} = 0, \text{and} \tag{A-5}$$

$$\frac{\partial\delta}{\partial v} = -\frac{(\lambda+b)k}{(a+v)^2 b}\log\frac{\lambda+b}{b} < 0. \tag{A-6}$$

These results in (A-5) and (A-6) lead to (1) and (2) of Lemma 1, respectively.

Proof of Proposition 2

From (6), the probability of detection in equilibrium is:

$$
\begin{aligned}
q(x^*) &= 1 - e^{-x^*} \\
&= 1 - \frac{b}{\lambda + b} \\
&= \frac{\lambda}{\lambda + b}.
\end{aligned}
$$

In addition, from (6), the manager's expected payoff, (1), is restated as:

$$
\begin{aligned}
EUM &= \left[\frac{b}{\lambda + b}(1 - \lambda + \delta\lambda) + \frac{\lambda}{\lambda + b}(1 - \lambda - b\delta)\right](1 + g)I \\
&= \frac{1}{\lambda + b}(b - b\lambda + b\delta\lambda + \lambda - \lambda^2 - b\delta\lambda)(1 + g)I \\
&= (1 - \lambda)(1 + g)I.
\end{aligned}
$$

Proof of the first-stage equilibrium condition

From the investor's expected payoff (2), the equilibrium condition (3), and the assumption $i = 0$:

$$
\begin{aligned}
I_M &= EUV \\
&= \left[\Big(1 - q(x)\Big)(1 - \delta)\lambda + q(x)\lambda\right](1 + g)I \\
&= \lambda\left[1 - \Big(1 - q(x)\Big)\delta\right](1 + g)I.
\end{aligned}
$$

Recall $I = I_M + W - F$, or $I_M = I - W + F$. Then:

$$
\begin{aligned}
I^* &= \frac{W - F^*}{1 - \lambda(1 + g)[1 - (1 - q(x^*))\delta^*]} \\
&= \frac{W - F^*}{1 - \lambda(1 + g)\Big(1 - e^{-x^*}\frac{kx^*e^{x^*}}{a + v}\Big)} \\
&= \frac{W - F^*}{1 - \lambda(1 + g)\Big(1 - \frac{kx^*}{a + v}\Big)}.
\end{aligned}
$$

About the audit fee and PA (4)

From the auditor's zero profit condition, (5), I can state the equilibrium audit fee as:

$$
\begin{aligned}
F^* &= C(x^*) + a\delta^*(1 - q(x^*)) - v\delta q(x^*) \\
&= \frac{k}{2}x^{*2} + a\frac{kx^*e^{x^*}}{a + v}e^{-x^*} - v\frac{kx^*e^{x^*}}{a + v}(1 - e^{-x^*}) \\
&= \frac{k}{2}x^{*2} + \frac{akx^*}{a + v} - v\frac{kx^*e^{x^*}}{a + v} + \frac{vkx^*}{a + v} \\
&= \frac{k}{2}x^{*2} + kx^* - \frac{vkx^*e^{x^*}}{a + v}.
\end{aligned}
$$

In equilibrium, the first derivative of $F(x)$ is:

$$\begin{aligned} F'(x) &= kx + k - \frac{vk}{a+v}(e^x + xe^x) \\ &= k(x+1)\left(1 - \frac{v}{a+v}e^x\right). \end{aligned}$$

Note $k > 0$ and $x > 0$. Then $F'(x) > 0$ holds if and only if $e^x < (a+v)/v$, or equivalently:

$$\lambda < \frac{ab}{v}. \tag{A-7}$$

I compare the audit fee, F, and the audit cost, C:

$$\begin{aligned} F - C(x) &= \frac{k}{2}x^2 + kx - \frac{vkxe^x}{a+v} - \frac{k}{2}x^2 \\ &= kx(1 - \frac{ve^x}{a+v}) \\ &= \frac{kvx}{a+v}(\frac{a+v}{v} - e^x). \end{aligned}$$

Hence, $F > C$ holds if, and only if, the inequality (A-7) holds. Because the manager chooses λ between 0 and 1, I can ensure that (A-7) always holds by assuming:

$$1 < \frac{ab}{v},$$

which constitutes PA (4) in Section 2.

Preparation for the comparative analysis

Recall that in the equilibrium of the game, the equality (11) holds and the optimal level of I is determined as in (10). I let D denote the denominator of (10):

$$D \equiv 1 - \lambda(1+g)(1 - \frac{kx}{a+v}).$$

Later, I prove that PAs ensure $D > 0$ and $I > 0$. These inequalities together ensure that the numerator of (10) is positive.

Next, I focus on the condition (11). Note:

$$\frac{dI}{d\lambda} = \frac{\partial I}{\partial \lambda} + \frac{\partial I}{\partial x}\frac{dx}{d\lambda}. \tag{A-8}$$

I consider the terms on the right hand side of (A-8): $\partial I/\partial\lambda$, $\partial I/\partial x$, and $dx/d\lambda$. First,

$$\begin{aligned} \frac{\partial I}{\partial \lambda} &= \frac{-\frac{\partial F}{\partial \lambda}D - \frac{\partial D}{\partial \lambda}(W-F)}{D^2} \\ &= \frac{-\frac{\partial D}{\partial \lambda}(W-F)}{D^2} \\ &= (1+g)(1 - \frac{kx}{a+v})\frac{W-F}{D^2}, \end{aligned}$$

where the second equality holds because $F(x)$ is not directly related to λ. In addition,

$$\frac{\partial I}{\partial x} = \frac{-\frac{\partial F}{\partial x}D - \frac{\partial D}{\partial x}(W-F)}{D^2}$$
$$= \frac{-\frac{\partial F}{\partial x}D - \lambda(1+g)\frac{k}{a+v}(W-F)}{D^2}, \text{ and}$$
$$\frac{dx}{d\lambda} = \frac{\frac{1}{\lambda+b}\frac{1}{b}}{\frac{1}{b}}$$
$$= \frac{1}{\lambda+b}.$$

Then, (11) can be restated as follows:

$$(1-\lambda)\left[\frac{(1+g)(1-\frac{kx}{a+v})(W-F)}{D^2} + \frac{1}{\lambda+b}\frac{-F'D-\lambda(1+g)\frac{k}{a+v}(W-F)}{D^2}\right] - \frac{W-F}{D} = 0. \quad \text{(A-9)}$$

Multiplying both sides of (A-9) by $D^2/(W-F) > 0$ yields:

$$(1-\lambda)\left[(1+g)\left(1-\frac{kx}{a+v}\right) + \frac{1}{\lambda+b}\left\{-\frac{F'D}{W-F} - \lambda(1+g)\frac{k}{a+v}\right\}\right] - D = 0. \quad \text{(A-10)}$$

Rearranging the left-hand side (LHS) of (A-10) yields:

$$\begin{aligned}
&(LHS)\\
&= (1-\lambda)(1+g)(1-\frac{kx}{a+v}) + \frac{1-\lambda}{\lambda+b}\left[-\frac{F'D}{W-F} - \lambda(1+g)\frac{k}{a+v}\right] - D\\
&= (1-\lambda)(1+g)(1-\frac{kx}{a+v}) - \frac{1-\lambda}{\lambda+b}\lambda(1+g)\frac{k}{a+v} - D - \frac{1-\lambda}{\lambda+b}\frac{F'D}{W-F}\\
&= (1-\lambda)(1+g)(1-\frac{kx}{a+v}) - \frac{1-\lambda}{\lambda+b}\lambda(1+g)\frac{k}{a+v}\\
&\quad -1 + \lambda(1+g)(1-\frac{kx}{a+v}) - \frac{1-\lambda}{\lambda+b}\frac{F'D}{W-F}\\
&= (1+g)(1-\frac{kx}{a+v}) - \frac{1-\lambda}{\lambda+b}\lambda(1+g)\frac{k}{a+v} - 1 - \frac{1-\lambda}{\lambda+b}\frac{F'D}{W-F}\\
&= g - (1+g)\frac{kx}{a+v} - \frac{1-\lambda}{\lambda+b}\lambda(1+g)\frac{k}{a+v} - \frac{1-\lambda}{\lambda+b}\frac{F'D}{W-F}\\
&= g - \frac{k(1+g)}{a+v}\left\{x + \lambda\frac{1-\lambda}{\lambda+b}\right\} - \frac{1-\lambda}{\lambda+b}\frac{F'D}{W-F}\\
&= g - \frac{k(1+g)}{a+v}\left\{x + \lambda\frac{1-\lambda}{\lambda+b}\right\} - \frac{1-\lambda}{\lambda+b}\frac{F'}{W-F}\left[1-\lambda(1+g)(1-\frac{kx}{a+v})\right]. \quad \text{(A-11)}
\end{aligned}$$

I let $H(\lambda)$ denote the expression in (A-11). Hence, the equilibrium condition (11) is equivalent to $H(\lambda)=0$.

A.2 Proofs of the Parameter Assumptions

I prove that PAs in Section 2 are sufficient for precluding corner solutions and hence for doing comparative analysis.

Lemma 2 *PA (1) to (3) in Section 2 constitute sufficient conditions for the following inequalities (1) to (4) to hold:*

1) $\delta^* < 1$

2) $\left[\dfrac{dEUM}{d\lambda}\right]_{\lambda=0} > 0$, *or equivalently,* $H(0) > 0$,

3) $\left[\dfrac{dEUM}{d\lambda}\right]_{\lambda=1} < 0$, *or equivalently,* $H(1) < 0$, *and*

4) $D > 0$.

Under (2) and (3) in Lemma 2, there is $\lambda \in (0,1)$ that satisfies the sufficient condition for EUM to take its local maximum:

$$\frac{dEUM}{d\lambda} = 0 \quad \text{and} \quad \frac{d^2EUM}{d\lambda^2} < 0. \tag{A-12}$$

Recall that this condition (A-12) can be restated as as:

$$H = 0 \quad \text{and} \quad \frac{dH}{d\lambda} < 0. \tag{A-13}$$

Furthermore, Proposition 2 ensures $I^* > 0$. If the manager chooses λ close to 0 in the first stage of the game, x^* and F^* approach 0 from (6) and (8). Even in this extreme case, I and EUM are positive from PA (4), (10) and (9). Hence, in his optimal choice, the manager chooses $I^* > 0$.

Proof of Lemma 2

To verify (1) in Lemma 2, recall (7). The equilibrium δ requires:

$$\delta^* = \frac{\lambda + b}{(a+v)b} k \log \frac{\lambda + b}{b} < 1.$$

By rearranging this inequality, I obtain:

$$k < \frac{(a+v)b}{(\lambda + b)\log \dfrac{\lambda + b}{b}}. \tag{A-14}$$

I later verify λ is in $(0,1)$. Hence, the inequality in (A-14) holds by restricting k as follows:

$$k < \frac{(a+v)b}{(1+b)\log \dfrac{1+b}{b}}.$$

This is PA (1) in Section 2.

To verify (2) in Lemma 2, I note:

$$H(0) = g - \frac{1}{b}\frac{k - \dfrac{vk}{a+v}}{W}.$$

Then, $H(0) > 0$ is equivalent to:

$$W > \frac{k}{bg}\frac{a}{a+v}.$$

This is PA(2) in Section 2.

To verify (3) and (4) in Lemma 2, I note:

$$\begin{aligned} H(1) &= g - \frac{k(1+g)}{a+v}x \\ &= -D(\lambda = 1). \end{aligned}$$

Hence:

$$H(1) < 0 \iff D(\lambda = 1) > 0. \tag{A-15}$$

In addition, I note:

$$D(\lambda = 0) = 1 > 0. \tag{A-16}$$

I consider the minimum of D for λ in $[0, 1]$.

[1] If D takes its minimum at λ in $(0, 1)$:

First, I note:

$$\begin{aligned} D &= 1 - \lambda(1+g)(1 - \frac{kx}{a+v}) \\ &= 1 - \lambda(1+g)(1 - \frac{k}{a+v}\log\frac{\lambda+b}{b}). \end{aligned} \tag{A-17}$$

The first and second derivatives of D with respect to λ are:

$$\begin{aligned} \frac{dD}{d\lambda} &= -(1+g)\left[(1 - \frac{k}{a+v}\log\frac{\lambda+b}{b}) + \lambda(-\frac{k}{a+v}\frac{b}{\lambda+b}\frac{1}{b})\right] \\ &= -(1+g)\left[1 - \frac{k}{a+v}\log\frac{\lambda+b}{b} - \frac{k}{a+v}\frac{\lambda}{\lambda+b}\right]. \\ \frac{d^2D}{d\lambda^2} &= -(1+g)\left[-\frac{k}{a+v}\frac{b}{\lambda+b}\frac{1}{b} - \frac{k}{a+v}\frac{(\lambda+b)-\lambda}{(\lambda+b)^2}\right] \\ &= -(1+g)\left[-\frac{k}{a+v}\frac{1}{\lambda+b} - \frac{k}{a+v}\frac{b}{(\lambda+b)^2}\right] \\ &= (1+g)\frac{k}{a+v}(\frac{1}{\lambda+b} + \frac{b}{(\lambda+b)^2}) > 0 \end{aligned} \tag{A-18}$$

I consider the minimum of D. If the first order condition is met, the expression in (A-18) is set equal to 0. Then:

$$1 - \frac{k}{a+v}\log\frac{\lambda+b}{b} = \frac{k}{a+v}\frac{\lambda}{\lambda+b}. \tag{A-19}$$

This equality (A-19) implies that the minimum of D in (A-17) is:

$$\begin{aligned} D_{min} &= 1 - \lambda(1+g)\frac{k}{a+v}\frac{\lambda}{\lambda+b} \\ &= 1 - \lambda^2(1+g)\frac{k}{a+v}\frac{1}{\lambda+b}. \end{aligned}$$

Then $D_{min} > 0$ is equivalent to:

$$1 > \lambda^2(1+g)\frac{k}{a+v}\frac{1}{\lambda+b}. \tag{A-20}$$

By rearranging (A-20), I obtain:

$$\begin{aligned} g &< \frac{1}{\lambda^2\frac{k}{a+v}\frac{1}{\lambda+b}} - 1 \\ &= (a+v)\frac{\lambda+b}{\lambda^2 k} - 1 \\ &= \frac{a+v}{k}(\frac{1}{\lambda} + \frac{b}{\lambda^2}) - 1. \end{aligned}$$

Hence, the condition:

$$\begin{aligned} g &< \frac{a+v}{k}(\frac{1}{1} + \frac{b}{1^2}) - 1 \\ &= \frac{(a+v)(b+1)}{k} - 1, \end{aligned}$$

ensures (4) in Lemma 2. Since D is always positive under this condition, (A-15) leads to $H(1) < 0$. Then, (3) in Lemma 2 holds.

[2] If D takes its minimum at $\lambda = 0$:

In this case, the inequality (A-16) implies $D > 0$ for any λ in $[0, 1]$. Hence, (4) in Lemma 2 holds. In addition, from (A-15), (3) in Lemma 2 is also true.

[3] If D takes its minimum at $\lambda = 1$:

$D(\lambda = 1) > 0$ suffices to ensure (4) in Lemma 2. In this case, (3) in Lemma 2 is immediate from (A-15). Hence, I impose:

$$D(\lambda = 1) = 1 - (1+g)(1 - \frac{k}{a+v}\log\frac{1+b}{b}) > 0. \tag{A-21}$$

Note that rearranging PA (1) gives:

$$\frac{k}{a+v}\log\frac{1+b}{b} < \frac{b}{1+b}. \tag{A-22}$$

This inequality (A-22) implies:

$$1 - \frac{k}{a+v}\log\frac{1+b}{b} > \frac{1}{1+b} > 0. \tag{A-23}$$

By rearranging the inequality in (A-21), I obtain:

$$g < \frac{1}{1 - \frac{k}{a+v}\log\frac{1+b}{b}} - 1. \tag{A-24}$$

From the discussion in [1] to [3], the constraints:

$$\begin{cases} g < \dfrac{(a+v)(b+1)}{k} - 1 & \text{(A-25)} \\ g < \dfrac{1}{1 - \dfrac{k}{a+v}\log\dfrac{1+b}{b}} - 1 & \text{(A-26)} \end{cases}$$

constitute PA (3) in Section 2.

A.3 Proofs of the Results in Section 4

Proof of Proposition 3

From the implicit function theorem, I obtain:

$$\frac{d\lambda}{dv} = -\frac{\frac{\partial H}{\partial v}}{\frac{\partial H}{\partial \lambda}}. \tag{A-27}$$

From the manager's second order condition, the following inequality holds (see [A-13], noting I use the different differential notations for clarity):

$$\frac{\partial H}{\partial \lambda} < 0. \tag{A-28}$$

In addition, from (A-11), I obtain:

$$\begin{aligned} \frac{\partial H}{\partial v} &= \frac{k(1+g)}{(a+v)^2}\left[x + \frac{\lambda(1-\lambda)}{\lambda+b}\right] \\ &\quad - \frac{1-\lambda}{\lambda+b}\left[\frac{\partial F'}{\partial v}\frac{D}{W-F} + \frac{F'D}{(W-F)^2}\frac{\partial F}{\partial v} + \frac{F'}{W-F}\frac{\partial D}{\partial v}\right], \end{aligned} \tag{A-29}$$

where:

$$\begin{aligned} \frac{\partial F'}{\partial v} &= -\left[k(a+v)^{-1} + vk(-1)(a+v)^{-2}\right](e^x + xe^x) \\ &= -k(e^x + xe^x)\frac{a+v-v}{(a+v)^2} \\ &= -\frac{ak(e^x + xe^x)}{(a+v)^2} < 0, \\ \frac{\partial F}{\partial v} &= -\frac{kxe^x(a+v) - vkxe^x}{(a+v)^2} \\ &= -\frac{akxe^x}{(a+v)^2} < 0, \end{aligned} \tag{A-30}$$

$$\frac{\partial D}{\partial v} = \lambda(1+g)(-1)\frac{kx}{(a+v)^2}$$
$$= -\frac{\lambda(1+g)kx}{(a+v)^2} < 0. \tag{A-31}$$

Hence, the expression (A-29) is positive. The total derivative in (A-27) is then positive.

Proof of Proposition 4

I express the derivative dI/dv as in (12). From (11), where I use the different differential notation for clarity, I obtain:

$$\frac{\partial I}{\partial \lambda} = \frac{I}{1-\lambda} > 0.$$

From Proposition 3, $d\lambda/dv$ is positive. In addition,

$$\frac{\partial I}{\partial v} = \frac{\frac{\partial (W-F)}{\partial v}D - (W-F)\frac{\partial D}{\partial v}}{D^2}$$
$$= \frac{-\frac{\partial F}{\partial v}D - (W-F)\frac{\partial D}{\partial v}}{D^2}, \tag{A-32}$$

where, from (A-30) and (A-31), the partial derivatives in the numerator of (A-32) are both negative.

Hence, the expression (A-32) is positive. The expression (12) is then positive.

Proof of Proposition 5

Differentiating x with respect to v shows:

$$\frac{dx}{dv} = \frac{d\log\frac{\lambda+b}{b}}{dv}$$
$$= \frac{b}{\lambda+b}\frac{d\frac{\lambda+b}{b}}{dv}$$
$$= \frac{b}{\lambda+b}\frac{\frac{d\lambda}{dv}}{b}$$
$$= \frac{\frac{d\lambda}{dv}}{\lambda+b} > 0,$$

where the final inequality follows from Proposition 3.

Proof of Proposition 6

Differentiating EUM with respect to v yields:

$$
\begin{aligned}
\frac{dEUM}{dv} &= (1+g)\left[-\frac{d\lambda}{dv}I + (1-\lambda)\frac{dI}{dv}\right] \\
&= (1+g)\left[-\frac{d\lambda}{dv}(1-\lambda)\frac{dI}{d\lambda} + (1-\lambda)\frac{dI}{dv}\right] \\
&= (1+g)(1-\lambda)\left(\frac{dI}{dv} - \frac{d\lambda}{dv}\frac{dI}{d\lambda}\right) \\
&= (1+g)(1-\lambda)\frac{\partial I}{\partial v} > 0,
\end{aligned}
$$

where the final inequality follows from (A-32).