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Weight Placed on the Rival's Profit under the Relative Performance Evaluation with Cournot-Bertrand Competition and Sequential Decision of the Weight

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Abstract

In this note, assuming a duopoly market, we revisit the weight placed on the rival's profit when two firms choose weights sequentially under Cournot-Bertrand competition. Based on the managerial delegation game, prior relative performance evaluation studies mainly focus on the optimal weight placed on the rival's profit and demonstrate that the sign of weights depends on economic environments. This study shows that if firms decide the quantity in a product market under Cournot-Bertrand competition, then the owner sets the positive weight on the rival's profit in managerial compensation contracts.

Keywords: relative performance evaluation; sequential game; Cournot-Bertrand competition; managerial delegation; managerial compensation contract

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1 Introduction

Several managerial compensation practices indicate the use of the relative performance evaluation (RPE). For example, in Japan, Mitsubishi Materials Corporation explicitly states the use of the RPE to compensate the manager.* Under the managerial delegation, it is important to consider optimal weights on peer performances because its incentive system may enhance the firm-wide performance by inducing the manager's behavior in the managerial accounting practice.

In RPE research, Hamamura (2021) demonstrates that the disadvantaged firm sets a positive weight on the rival's profit under quantity competition. This is one of the important results because seminal works show that if firms face quantity competition in the product market, then owners choose the negative weight for strategic substitutes (Aggarwal and Samwick 1999; Fumas 1992). Additionally, focusing on industrial profits, several firms cooperate to facilitate the product market in practice. In RPE research, as a disadvantaged firm, Hamamura (2021) considers two cases, the inefficient marginal cost firm and the follower. Actually, in practice, we can observe many examples of both cases.

Following Hamamura (2021), Hamamura (2022) analyzes the optimal level of weights under asymmetric costs and Cournot-Bertrand competition. In practice, it is difficult to distinguish which firms set the price or quantity in a product market, and therefore, in several cases, firms face Cournot-Bertrand competition in a product market. However, Hamamura (2022) does not examine the other disadvantage case.† In other words, previous studies do not consider the sequential game under Cournot-Bertrand competition to specify the optimal weight placed on the rival's profit under accounting-based RPEs. Therefore, supposing RPEs to compensate managers, we revisit the optimal weight placed on the rival's profit when two firms choose weights sequentially under Cournot-Bertrand competition.

Let us summarize our results. If the leader in the sequential game decides the quantity and the follower decides the price, the leader sets the positive weight, and the follower sets the negative weight on the rival's profit. On the other hand, if the leader in the sequential game decides the price and the follower decides the quantity, then the leader sets the negative weight, and the follower sets the positive weight on the rival's profit under the RPE.

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* We can observe this information on managerial compensation contracts in the Integration Report of Mitsubishi Materials Corporation. URL: https://ssl4.eir-parts.net/doc/5711/ir_material_for_fiscal_ym7/121391/00.pdf (Last accessed on September 4, 2023)

† Xu and Matsumura (2022) also considers Cournot-Bertrand competition with convex costs and RPEs. However, they do not consider the sequential game. Additionally, because, while Hamamura (2021) supposes quantity competition, our paper examines Cournot-Bertrand competition, we do not call the follower as a disadvantaged firm in this paper.

2 Model

Firms 1 and 2 engage in product market competition. The profit function of firm i $(=1,\ 2)$ is $\pi_i=(p_i-c_i)\,q_i$, where p_i represents the market price, c_i is the marginal cost, and q_i denotes the quantity of firm i. We assume $c_1=c_2=0$ for simplicity. Owners compensate managers using profit-based RPEs. Thus, we assume

$$V_i=\pi_i+\gamma_i\pi_j,\ i,j=1,2,\ i\neq j, \quad (1)$$

where V_i is the payoff function of firm i's manager, and γ_i represents the weight assigned to the rival's profit. Managers decide on the quantity or price in a duopoly market to maximize Eqs. (1). Owners choose the level of γ_i to maximize their profits, π_i.

The timeline of events is as follows. First, the owner of Firm 1 decides the level of γ_1 in Stage 1. Second, the owner of Firm 2 decides the level of γ_2 in Stage 2. Lastly, the manager chooses the quantity or price in Stage 3.

3 Results

We identify the sub-game perfect Nash equilibrium (SPNE) by backward induction, considering two cases: (i) Firm 1 decides the quantity, and Firm 2 decides the price, and (ii) Firm 1 decides the price, and Firm 2 decides the quantity.

3.1 Firm 1 Decides the Quantity, and Firm 2 Decides the Price

We suppose the following demand functions.[‡]

$$p_1^{(Q,P)}=a\,(1-\theta)-\left(1-\theta^2\right)q_1+\theta\ p_2,\qquad q_2^{(Q,P)}=a-p_2-\theta\ q_1,$$

where a represents the constant greater than zero, $\theta\in(0,1)$ denotes the degree of product substitution, and superscript (Q,P) indicates the case in which Firm 1 decides the quantity and Firm 2 decides the price in the product market.

In Stage 3, from the first-order condition (FOC), we obtain the following best response functions.[§]

[‡] Following earlier studies (e.g., Singh and Vives 1984), we define the utility function of a representative consumer as $U=a\,(q_1+q_2)-(q_1^2+2\theta\ q_1q_2+q_2^2)/2$ where $a>c$ and $\theta\in(0,\ 1)$. Using this utility function and the budget constraint, we consider $\max_{q_1,\ q_2}\ U-p_1q_1-p_2q_2$. From this analysis, we obtain inverse demand functions which are represented as $p_i=a-q_i-\theta\,q_j$. Rearranging the inverse demand function, we obtain the demand functions under Cournot-Bertrand competition. Tremblay and Tremblay (2011) is a typical paper to derive this type of the demand functions.

[§] We can confirm that the second order condition is satisfied in this case because we get $\partial^2V_1/\partial q_1^2=-2(1-\theta)(1+\theta)<0$ and $\partial^2V_2/p_2^2=-2<0$.

$$BR_1^{(Q,P)}(p_2) = \frac{a(1-\theta)+(1-\gamma_1)\,\theta p_2^{(Q,P)}}{2(1-\theta)(1+\theta)},\ BR_2^{(Q,P)}(q_1) = \frac{a-(1-\gamma_2)\,\theta q_1^{(Q,P)}}{2}.$$

One can notice that, under $\gamma_i < 1$, q_1 increases as p_2 increases, and p_2 decreases as q_1 increases. This outcome indicates that it is important to consider the rival's decision variable in a product market for strategic relationships. In other words, q_1 is strategic complements to p_2, and p_2 is strategic substitutes to q_1. When we examine the weight placed on the rival's profit, its strategic relationship plays an important role. Additionally, if Firm 1 rises γ_1, then q_1 decreases and p_2 increases. On the other hand, if Firm 2 enhances γ_2, then p_2 increases and q_1 increases. Therefore, Firm 1 has the incentive to raise γ_1, and Firm 2 has the incentive to reduce γ_2. Using the above outcomes, we obtain the following outcomes in this stage.

$$q_1^{(Q,P)} = \frac{a\left[2-(1+\gamma_1)\,\theta\right]}{4-\theta^2\left[3+\gamma_1\,(1-\gamma_2)+\gamma_2\right]},\ p_2^{(Q,P)} = \frac{a(1-\theta)\left[2+(1+\gamma_2)\,\theta\right]}{4-\theta^2\left[3+\gamma_1\,(1-\gamma_2)+\gamma_2\right]}.$$

Next, we analyze the optimal decision by the follower in Stage 2. From the FOC, Firm 2 sets the following weight.

$$\gamma_2^{(Q,P)} = -\frac{(1-\gamma_1)\,\theta\,(2+\theta)}{4+2\,(1-\gamma_1)\,\theta-(1+3\gamma_1)\,\theta^2}.$$

Because $\partial\gamma_2^{(Q,P)}/\partial\gamma_1 = 4\theta(2+\theta)\left(1-\theta^2\right)/\left[4+2\,(1-\gamma_1)\,\theta-(1+3\gamma_1)\,\theta^2\right]^2 > 0$, if Firm 1 raises γ_1, then Firm 2 also enhances γ_2. Therefore, to enhance Firm 2's price for tacit collusion, Firm 1 sets the large γ_1 for strategic complements. Using this outcome, we identify $\gamma_1^{(Q,P)}$ and summarize the result as follows.

Result 1 *When the leader decides the quantity and the follower decides the price in a product market, we obtain the following outcomes.*

$$\gamma_1^{(Q,P)} = \frac{\left(4+2\theta-\theta^2\right)\theta}{4+2\theta-2\theta^2+\theta^3},\ \gamma_2^{(Q,P)} = -\frac{(2-\theta)\,\theta}{4+2\theta-\theta^2},$$

$$q_1^{(Q,P)} = \frac{a\left(4+2\theta-\theta^2\right)}{8\,(1+\theta)},\ q_2^{(Q,P)} = \frac{a\,(2+\theta)}{4\,(1+\theta)},$$

$$p_1^{(Q,P)} = \frac{a\left(4+2\theta-\theta^2\right)}{8\,(1+\theta)},\ p_2^{(Q,P)} = \frac{a\left(4+2\theta-2\theta^2+\theta^3\right)}{8\,(1+\theta)},$$

$$\pi_1^{(Q,P)} = \frac{a^2\left(4+2\theta-\theta^2\right)^2}{64(1+\theta)^2},\ \pi_2^{(Q,P)} = \frac{a^2(2+\theta)\left(4+2\theta-2\theta^2+\theta^3\right)}{32(1+\theta)^2}.$$

All outcomes are positive, and the second order condition is satisfied under $0 < \theta < 1$.

We discuss the sign of $\gamma_i^{(Q,P)}$. Because we assume $0 < \theta < 1$, we obtain $\gamma_1^{(Q,P)} > 0$ and $\gamma_2^{(Q,P)} < 0$ straightforwardly. We summarize this result in the following proposition.

Proposition 1 *When duopoly firms face Cournot-Bertrand competition under the sequential game, if the leader decides the quantity, then the leader sets the positive weight, and the follower sets the negative weight on the rival's profit under the RPE.*

Classical studies (Aggarwal and Samwick 1999; Fumas 1992) demonstrate that when the firm decides the strategic complementary variable in the product market, the firm sets the positive weight. On the other hand, when the firm decides the strategic substitute variable in the product market, the firm sets the negative weight. Our result may indicate that, under Cournot-Bertrand competition, it does not play an important role in determining whether the sign of the weight is positive or negative.

3.2 Firm 1 Decides the Price, and Firm 2 Decides the Quantity

Next, we consider the case in which Firm 1 (leader) decides the price, and Firm 2 (follower) decides the quantity. In this case, we suppose the following demand functions.

$$q_1^{(P,Q)} = a - p_1 - \theta\, q_2, \qquad p_2^{(P,Q)} = a\,(1-\theta) - \left(1-\theta^2\right) q_2 + \theta\, p_1,$$

where superscript (P, Q) represents the case in which Firm 1 decides the price and Firm 2 decides the quantity in the product market.

In Stage 3, considering the FOC, we obtain

$$BR_1^{(P,Q)}\,(q_2) = \frac{a - (1-\gamma_1)\,\theta q_2^{(P,Q)}}{2}, \quad BR_2^{(P,Q)}\,(p_1) = \frac{a(1-\theta) + (1-\gamma_2)\,\theta p_1^{(P,Q)}}{2(1-\theta)(1+\theta)}.$$

Compared with the previous analysis, this is a reversal outcome, and the effect of γ_1 on firms is different. In other words, the strategic effects are different from the previous case.

Using the above outcome, we explore the optimal γ_2 in Stage 2. From the FOC, we get

$$\gamma_2^{(P,Q)} = \frac{(1-\gamma_1)\,(2-\theta)\theta}{4 - 2\,(1-\gamma_1)\,\theta - (1+3\gamma_1)\,\theta^2}.$$

Because, under $\gamma_1 < 1$, we obtain $\partial\gamma_2^{(P,Q)}/\partial\gamma_1^{(P,Q)} = -4(2-\theta)\theta\left(1-\theta^2\right) / \left[4 - 2\,(1-\gamma_1)\,\theta + (1 + 3\gamma_1)\,\theta^2\right]^2 < 0$, we can confirm that, contrarily to the previous section, γ_1 has a negative impact on γ_2. This outcome implies that, to reduce Firm 2's quantity for credible threaten, Firm 1 sets the small γ_1 for a strategic substitute. We identify $\gamma_1^{(P,Q)}$ and summarize the result as follows.

Result 2 *When the leader decides the price and the follower decides the quantity in a product market, then we obtain the following outcomes.*

$$\gamma_1^{(P,Q)} = -\frac{\theta\left(4-2\theta-\theta^2\right)}{4-2\theta-2\theta^2-\theta^3},\ \gamma_2^{(P,Q)} = \frac{\theta(2+\theta)}{4-2\theta-\theta^2},$$
$$q_1^{(P,Q)} = \frac{a\left(4-2\theta-\theta^2\right)}{8(1-\theta)(1+\theta)},\ q_2^{(P,Q)} = \frac{a\left(4-2\theta-2\theta^2-\theta^3\right)}{8(1-\theta)(1+\theta)},$$
$$p_1^{(P,Q)} = \frac{a\left(4-2\theta-\theta^2\right)}{8},\ p_2^{(P,Q)} = \frac{a(2-\theta)}{4},$$
$$\pi_1^{(P,Q)} = \frac{a^2\left(4-2\theta-\theta^2\right)^2}{64(1-\theta)(1+\theta)},\ \pi_2^{(P,Q)} = \frac{a^2(2-\theta)\left(4-2\theta-2\theta^2-\theta^3\right)}{32(1-\theta)(1+\theta)}.$$

All outcomes are positive, and second order condition is satisfied, when $4-2\theta-2\theta^2-\theta^3>0$ *is satisfied.*

Using the above outcomes, we specify the sing of $\gamma_i^{(P,Q)}$. Under the positive conditions, we can get $\gamma_1^{(P,Q)}<0$ and $\gamma_2^{(P,Q)}>0$ straightforwardly because $4-2\theta-\theta^2>0$. From this outcome, we conduct the following proposition.

Proposition 2 *When duopoly firms face Cournot-Bertrand competition under the sequential game, if the leader decides the price, the leader sets the negative weight, and the follower sets the positive weight on the rival's profit under RPEs.*

This is a just reversal outcome of Proposition 1. In other words, the decision timing does not play an important role, and the strategic relationship has a significant effect on the weight.

4 Discussion

This note documents the optimal weight placed on the rival's profit under the RPE with the sequential game and Cournot-Bertrand competition. If the leader decides the quantity, the leader sets the positive weight, and the follower sets the negative weight on the rival's profit. In contrast, if the leader decides the price, the leader sets the negative weight, and the follower sets the positive weight on the rival's profit.

According to Hamamura (2021, 2022), one can anticipate that we obtain the analogies of Hamamura (2022). That is, depending on parameters, both firms set the positive weight placed on the rival's profit in a specific case. However, our result demonstrates that, in our assumption, one firm only sets the positive weight. This result implies that while the strategic effect has a significant role, the decision timing does not affect the weight. In particular, while, according to Hamamura

(2022), the difference in marginal costs is the most important factor to decide the weight, our paper does not consider the continuous variable. In other words, the difference between firms is represented as two discrete variables, and the effect of its difference is fixed in our model. One can consider the reason for differences between our study and Hamamura (2022).

Our analysis may help to understand the managerial compensation contract in practice as a main contribution. According to Hamamura and Inoue (2023), empirical results suggest that Japanese firms set peer compensation which depends on peer profits as a benchmark. This means that if the rival earns a large profit, then the manager obtains a large compensation. However, several firms apply the RPE to reward the manager. Our model explains this practice which is the asymmetric rewarding system.

In future research, one may consider the asymmetric performance evaluation system proposed by Hamamura and Ramani (2023), because, recently, many firms consider social performance to evaluate the manager in practice.

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